

Dark Energy From Fifth Dimension

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Observational evidence for the existence of dark energy is strong. Here we suggest a model which is based on a modified gravitational theory in 5D and interpret the 5th dimension as a manifestation of dark energy in the 4D observable universe. We also obtain an equation of state parameter which varies with time. Finally, we match our model with observations by choosing the free parameters of the model.

Keywords: dark energy, extra dimensions, Kaluza-Klein theory.

I. INTRODUCTION

Recent observations suggest that the Universe is not only expanding, but also accelerating [1, 2]. The first candidate for dark energy was a cosmological constant. It was originally introduced by Einstein in 1917 to achieve a static universe. However, after Hubble observations which suggested that the universe is not static, it was abandoned.

In particle physics, the cosmological constant normally arises as an energy density of the vacuum. The energy scale of the cosmological constant (Λ) should be much larger than that of the present Hubble constant H_0 , if it originates from the vacuum energy density. This is the cosmological constant problem.

In addition to the cosmological constant proposal, there are a lot of alternative routes which have been proposed to explain the accelerated expansion of the Universe. Some of the most important are as follows [3]:

- 1-Quintessence models[4];
- 2-Scalar field models[5];
- 3-Chameleon fields [6];
- 4-K-essence [7];
- 5-Modified gravity arising out of string theory [8] or generalization of GR [9, 10];
- 6-Phantom dark energy [11];
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The Einstein field equations consist of two parts. The first part contains the geometry and the second part contains the matter. Some of dark energy (DE) models include the cosmological constant and a scalar field which modify the rhs by introducing some extra terms in it. There is another way: modifying the geometry (i.e. the lhs of the Einstein's equations). The geometrical modifications can arise from quantum effects such as higher curvature corrections to the Einstein-Hilbert (EH) action. In [12] by introducing a quadratic term in R , an inflationary solution in the early universe was obtained. However, it was pointed out in [9, 10] that the late time acceleration can be realized by adding a term containing inverse power of Ricci scalar to the EH action. The structure of this paper is as follows: in section II, we review the modified gravity theories. In section III, we discuss briefly the Kaluza-Klein gravity and the induced matter in this theory. In section IV, we apply the STM formalism in CDTT model. Then in section V, we extract energy density and the pressure of dark energy from STM formalism. Finally, in section VI, we will match our model with observations by choosing the free parameters of the model.

II. MODIFIED GRAVITY

We first investigate a fairly general way of modifying gravitational theory which is known as $f(R)$ gravity. The formalism starts with the introduction of an action in the form:

$$S = \int \sqrt{-g} f(R) d^4x + \int \sqrt{-g} \mathcal{L}_M d^4x \quad (1)$$

where $f(R)$ is an arbitrary function of R such as $R + \alpha R^2$, $R + \alpha \ln R$, $R + \frac{\alpha}{R}$ etc, and \mathcal{L}_M is a matter Lagrangian density. By varying the action (1) with respect to the metric $g_{\mu\nu}$, one obtains the following field equations[10]:

$$\frac{1}{2} g_{\mu\nu} f(R) - R_{\mu\nu} f'(R) - \nabla_\mu \nabla_\nu f'(R) - g_{\mu\nu} \nabla^2 f'(R) + \frac{1}{2} T_{\mu\nu} = 0 \quad (2)$$

where $\mu, \nu = 0 \dots 3$ and a prime shows differentiate with respect to R . By taking the trace of (3) in the absence of matter and constant curvature ($\nabla_\alpha R = 0$) case, we find:

$$f(R) - \frac{1}{2} R f'(R) = 0 \quad (3)$$

In [9] Carroll et.al. considered a $f(R)$ function of the form (CDTT model):

$$f(R) = R - \frac{\mu^4}{R} \quad (4)$$

where μ is a new parameter with dimension of $[time]^{-1}$. By means of (3), we obtain the following field equations for CDTT model:

$$\frac{1}{2} g_{\mu\nu} (R - \frac{\mu^4}{R}) - R_{\mu\nu} (1 + \frac{\mu^4}{R^2}) - \mu^4 (\nabla_\mu \nabla_\nu + g_{\mu\nu} \nabla^2) \frac{1}{R^2} + \frac{1}{2} T_{\mu\nu} = 0 \quad (5)$$

The constant-curvature vacuum solutions in 4D, for which $\nabla_\mu R = 0$, satisfy $R = \pm\sqrt{3}\mu^2$. Thus one finds the interesting result that the constant curvature vacuum solutions in 4D are not Minkowski space but rather are de Sitter or anti de Sitter spaces. Moreover in spherically symmetric case (the black hole solutions), one obtains a Schwartzchild-de Sitter black hole.

By considering a perfect fluid case i.e:

$$T_{\mu\nu}^M = (\rho_M + p_M)U_\mu U_\nu + p_M g_{\mu\nu} \quad (6)$$

where U^μ is the fluid rest-frame four velocity, ρ_M is the energy density, p_M is the pressure and we consider an equation of state of the form $p_M = \omega\rho_M$, also we take the metric of the flat Robertson-Walker (in four dimension) form, i.e. $ds^2 = -dt^2 + a(t)^2 dx^2$. One obtains the following equation:

$$3H^2 - \frac{\mu^4}{12(\dot{H} + 2H^2)^2}(2H\ddot{H} + 15H^2\dot{H} + 2\dot{H}^2 + 6H^4) = \rho_M \quad (7)$$

for the time-time component of the field equation and:

$$\dot{H} + \frac{3}{2} - \frac{\mu^4}{12(\dot{H} + 2H^2)^2}(4\dot{H} + 9H^2 - a^2\partial_0\partial_0(\frac{1}{a^2}) - 2a^2H\partial_0(\frac{1}{a^2})) = -\frac{1}{2}p_M \quad (8)$$

for the space-space component. Equation (7) reduces to the usual Friedman equation by setting $\mu = 0$. In [9] Carroll et.al. obtained some solutions such as eternal de Sitter, power law acceleration and future singularity in vacuum case. As mentioned in the Introduction, the solutions of this theory contain instabilities [13].

In [14] Dolgove and Kawasaki also find instability in the matter section of the above modified gravity model. In [15] Faraoni suggested that generally a model is stable if it satisfies the condition: $f'' > 0$ and is unstable if $f'' < 0$.

Dicke in [16] discussed the Newtonian limit in singular nonlinear modified gravity models (i.e. $f(0) = \infty$) and found that a model has Newtonian limit if it satisfies $f''(R_0) = 0$ where R_0 is the solution of (3). Therefore the $f(R)$ model which is introduced in (4), hasn't Newtonian limit and is unstable.

III. KALUZA-KLEIN GRAVITY AND INDUCED MATTER

Kaluza unified electromagnetism with gravity in 1921 by applying Einstein's general theory of relativity to a five rather than four-dimensional spacetime manifold [17]. The Einstein equations, in five dimensions with no five dimensional energy-momentum tensor, are:

$$\hat{G}_{AB} = 0 \quad (9)$$

or equivalently:

$$\hat{R}_{AB} = 0 \quad (10)$$

where hat denotes a five dimensional quantity and $A, B = 0 \dots 4$ and $\hat{G}_{AB} \equiv \hat{R}_{AB} - \frac{1}{2}\hat{R}\hat{g}_{AB}$ is the Einstein tensor, \hat{R}_{AB} and $\hat{R} = \hat{g}_{AB}\hat{R}^{AB}$ are the five dimensional Ricci tensor and Ricci scalar and \hat{g}_{AB} is the five dimensional metric. The absence of matter in the five dimensional universe is the Einstein's idea which says that the universe in higher dimensions is empty.

The five dimensional Ricci tensor and Christoffel symbols are defined in terms of metric exactly as in four dimensions (minimal extension). Then everything depends on one's choice for the form of the five dimensional metric. Kaluza proposed the following metric:

$$\hat{g}_{AB} = \begin{pmatrix} g_{\alpha\beta} + \kappa^2\phi^2 A_\alpha A_\beta & \kappa\phi^2 A_\alpha \\ \kappa\phi^2 A_\beta & \phi^2 \end{pmatrix} \quad (11)$$

where the electromagnetic potential is scaled by a constant κ in order to get the right multiplicative factor in action.

There is an important question with the Kaluza's assumption: where is the fifth dimension? Why do we not observe it? Kaluza proposed the cylindrical condition to overcome this problem which means dropping all derivatives with respect to the fifth coordinate.

In 1926 Klein showed that Kaluza's cylinder condition would arise naturally if the fifth dimension has (1) a circular topology, in which case physical fields would depend on it only periodically, and could be Fourier-expanded; and (2) a small enough ("compactified") scale in which case the energies of all Fourier modes above the ground state could be made so high as to be unobservable. This version of Klein theory is called compactified Kaluza-Klein gravity.

If one applies the cylinder condition and use the metric (11) and field equations (9), then one will find that the $\alpha\beta$ -, $\alpha 4$ - and 44 -components of the five dimensional field equations (9) reduce respectively to the following field equations in four dimensions:

$$\begin{aligned} G_{\alpha\beta} &= \frac{\kappa^2\phi^2}{2}T_{\alpha\beta}^{EM} - \frac{1}{\phi}[\nabla_\alpha(\partial_\beta\phi) - g_{\alpha\beta}\square\phi] \\ \nabla^\alpha F_{\alpha\beta} &= -3\frac{\partial^\alpha\phi}{\phi}F_{\alpha\beta} \\ \square\phi &= \frac{\kappa^2\phi^3}{4}F_{\alpha\beta}F^{\alpha\beta} \end{aligned} \quad (12)$$

where $G_{\alpha\beta} \equiv R_{\alpha\beta} - Rg_{\alpha\beta}/2$ is the Einstein tensor in four dimensions, $T_{\alpha\beta}^{EM} \equiv g_{\alpha\beta}F_{\gamma\delta}\frac{F^{\gamma\delta}}{4} - F_{\alpha}^{\gamma}F_{\beta\gamma}$ is the electromagnetic energy-momentum tensor, and $F_{\alpha\beta} \equiv \partial_{\alpha}A_{\beta} - \partial_{\beta}A_{\alpha}$, where $\alpha, \beta = 0 \dots 3$. If the scalar field ϕ is constant throughout spacetime, then the first two equations of (12) are just the Einstein and Maxwell equations :

$$G_{\alpha\beta} = 8\pi G\phi^2 T_{\alpha\beta}^{EM}$$

$$\nabla^{\alpha}F_{\alpha\beta} = 0 \tag{13}$$

where we have identified the scaling parameter κ in terms of the gravitational constant G (in four dimensions) by:

$$\kappa \equiv 4\sqrt{\pi G} \tag{14}$$

This is the result originally obtained by Kaluza and Klein, who set $\phi = 1$. The condition $\phi = \text{constant}$. is consistent with the third equation of (12) when $F_{\alpha\beta}F^{\alpha\beta} = 0$, which was first pointed out by Jordan [18].

An alternative is to abandon the cylindrical condition [19, 20]. Therefore the metric depends on the fifth dimension and this dependence allows one to obtain electromagnetic radiation, dust and other forms of cosmological matter. These types of theories are called noncompactified Kaluza-Klein theories.

Wesson proposed that the fifth coordinate ψ might be related to the rest mass. Dimensionally $x^4 = \frac{Gm}{c^2}$ allows us to treat the rest mass m of a particle as a length coordinate, in analogy with $x^0 = ct$. We can say that the four-dimensional matter is a manifestation of the five-dimensional geometry [19].

In noncompactified Kaluza-Klein theories we begin with a metric of the form:

$$\hat{g}_{AB} = \begin{pmatrix} g_{\alpha\beta} & 0 \\ 0 & \epsilon\phi^2 \end{pmatrix} \tag{15}$$

where we have introduced the factor ϵ in order to allow a timelike or a spacelike signature for the fifth dimension (and $\epsilon^2 = 1$).

Now by using the five dimensional field equations (9) in vacuum and keeping derivatives with respect to the fifth coordinate x^4 , the resulting expression for the $\alpha\beta$ - $\alpha 4$ - and 44 - parts of the

five dimensional Ricci tensor $R_{\alpha\beta}$ are :

$$\begin{aligned}\hat{R}_{\alpha\beta} = & R_{\alpha\beta} - \frac{\nabla_\beta(\partial_\alpha\phi)}{\phi} + \frac{\epsilon}{2\phi^2} \left(\frac{\partial_4\phi\partial_4g_{\alpha\beta}}{\phi} - \partial_4g_{\alpha\beta} \right. \\ & \left. + g^{\gamma\delta}\partial_4g_{\alpha\gamma}\partial_4g_{\beta\delta} - \frac{g^{\gamma\delta}\partial_4g_{\gamma\delta}\partial_4g_{\alpha\beta}}{2} \right)\end{aligned}\quad (16)$$

$$\begin{aligned}\hat{R}_{\alpha 4} = & \frac{g^{44}g^{\beta\gamma}}{4}(\partial_4g_{\beta\gamma}\partial_4g_{44} - \partial_\gamma g_{44}\partial_4g_{\alpha\beta}) + \frac{\partial_\beta g^{\beta\gamma}\partial_4g_{\gamma\alpha}}{2} \\ & + \frac{g^{\beta\gamma}\partial_4(\partial_\beta g_{\gamma\alpha})}{2} - \frac{\partial_\alpha g^{\beta\gamma}\partial_4(g_{\beta\gamma})}{2} - \frac{g^{\beta\gamma}\partial_4\partial_\alpha g_{\beta\gamma}}{2} \\ & + \frac{g^{\beta\gamma}g^{\delta\epsilon}\partial_4g_{\gamma\alpha}\partial_\beta g_{\delta\beta}}{4} + \frac{\partial_4g^{\beta\gamma}\partial_\alpha g_{\beta\gamma}}{4}\end{aligned}\quad (17)$$

$$\begin{aligned}\hat{R}_{44} = & -\epsilon\phi\Box\phi - \frac{\partial_4g^{\alpha\beta}\partial_4g_{\alpha\beta}}{2} - \frac{g^{\alpha\beta}\partial_4(\partial_\alpha\beta)}{2} + \frac{\partial_4\phi g^{\alpha\beta}\partial_4g_{\alpha\beta}}{2\phi} \\ & - \frac{g^{\alpha\beta}g^{\gamma\delta}\partial_4g_{\gamma\beta}\partial_4g_{\alpha\delta}}{4}\end{aligned}\quad (18)$$

Then Eq. (16) gives the following expression for the four dimensional Ricci tensor:

$$\begin{aligned}R_{\alpha\beta} = & \frac{\nabla_\beta(\partial_\alpha\phi)}{\phi} - \frac{\epsilon}{2\phi^2} \left[\frac{\partial_4\phi\partial_4g_{\alpha\beta}}{\phi} - \partial_4(\partial_4g_{\alpha\beta}) \right. \\ & \left. + g^{\gamma\delta}\partial_4g_{\alpha\gamma}\partial_4g_{\beta\delta} - \frac{g^{\gamma\delta}\partial_4g_{\gamma\delta}\partial_4g_{\alpha\beta}}{2} \right]\end{aligned}\quad (19)$$

The above equation allows us to interpret the four dimensional matter as a manifestation of the five dimensional geometry. We assume that the Einstein Field equations hold in four dimensions i.e.:

$$8\pi GT_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2}Rg_{\alpha\beta} \quad (20)$$

where $T_{\alpha\beta}$ is the four-dimensional matter energy momentum tensor. By contracting (19) with $g_{\alpha\beta}$, we obtain the following expression for Ricci scalar:

$$R = \frac{\epsilon}{4\phi^4} [\partial_4g^{\alpha\beta}\partial_4g_{\alpha\beta} + (g^{\alpha\beta}\partial_4g_{\alpha\beta})^2] \quad (21)$$

inserting (21) and (19) into (20) one finds:

$$\begin{aligned}8\pi GT_{\mu\nu} = & \frac{\nabla_\beta(\partial_\alpha\phi)}{\phi} - \frac{\epsilon}{2\phi^2} \left[\frac{\partial_4\phi\partial_4g_{\alpha\beta}}{\phi} - \partial_4(\partial_4g_{\alpha\beta}) + g^{\gamma\delta}\partial_4g_{\alpha\gamma}\partial_4g_{\beta\gamma} \right. \\ & \left. - \frac{g^{\alpha\gamma}\partial_4g_{\gamma\delta}\partial_4g_{\alpha\beta}}{2} + \frac{g_{\alpha\beta}}{4}(\partial_4g^{\gamma\delta}\partial_4g_{\gamma\delta} + (g^{\gamma\delta}\partial_4g_{\gamma\delta})^2) \right]\end{aligned}\quad (22)$$

If we use this expression for $T_{\mu\nu}$, the four-dimensional Einstein equations $G_{\alpha\beta} = 8\pi GT_{\alpha\beta}$ are contained in the five-dimensional vacuum ones $\hat{G}_{AB} = 0$. The matter described by $T_{\mu\nu}$ is a manifestation of pure geometry in the five dimensional world. There are solutions for different types of metric and energy-momentum tensor, such as the spherically symmetric case [20], the isotropic and homogenous case [21], etc.

IV. 5D MODIFIED GRAVITY IN STM FORMALISM

It is easy to check that all $f(R)$ gravity theories in the vacuum and constant curvature case are equivalent to the Einstein field equations in the presence of a cosmological constant, so we can use any $f(R)$ model. In the constant curvature and no matter case, Eq.(3) reduces to :

$$R_{\mu\nu}f'(R) - \frac{1}{2}g_{\mu\nu}f(R) = 0 \quad (23)$$

From (4) we obtain R_0 and by substituting it in $f(R)$ and its derivative, and then by replacing it in (23), the following relation is obtained:

$$R_{\mu\nu} + \beta(R_0)g_{\mu\nu} = 0 \quad (24)$$

where:

$$\beta(R_0) = -\frac{f(R_0)}{f'(R_0)} \quad (25)$$

which can be rewritten as Einstein field equation in the presence of a cosmological constant, where:

$$\Lambda = \beta + \frac{R_0}{2} \quad (26)$$

In this section we use the CDTT model and work with a 5D extension of the flat RW metric in the form:

$$ds^2 = dt^2 - a(t)^2(dr^2 + r^2d\Omega^2) - R(t)^2d\psi^2 \quad (27)$$

where $a(t)$ is the scale factor of ordinary 3D spatial dimensions and $R(t)$ is the scale factor of the fifth dimension. In five dimensions, the solution of (3) for CDTT model is (with $M_P \equiv (8\pi G)^{-\frac{1}{2}} = 1$):

$$\hat{R} = \pm\sqrt{\frac{7}{3}}\mu^2 \quad (28)$$

Here we choose the minus sign.

Now we try to find out $a(t)$ and $R(t)$. By replacing (27) and (28) in (23) in five dimensional and vacuum (STM formalism) and constant curvature case i.e:

$$H_{AB} = (1 + \frac{\mu^4}{\hat{R}^2})\hat{R}_{AB} - \frac{1}{2}(1 - \frac{\mu^4}{\hat{R}^2})\hat{R}\hat{g}_{AB} = 0 \quad (29)$$

we obtain the following equations:

$$H_0^0 = -15\frac{\ddot{a}(t)}{a(t)} - 5\frac{\ddot{R}(t)}{R(t)} + \frac{\sqrt{21}}{3}\mu^2 = 0 \quad (30)$$

$$H_1^1 = H_2^2 = H_3^3 = -5\frac{\ddot{a}(t)}{a(t)} - 10\left(\frac{\dot{a}(t)}{a(t)}\right)^2 - 5\frac{\dot{a}(t)}{a(t)}\frac{\dot{R}(t)}{R(t)} + \frac{\sqrt{21}}{3}\mu^2 = 0 \quad (31)$$

$$H_4^4 = -5\frac{\ddot{R}(t)}{R(t)} - 15\frac{\dot{a}(t)}{a(t)}\frac{\dot{R}(t)}{R(t)} + \frac{\sqrt{21}}{3}\mu^2 = 0 \quad (32)$$

By computing \hat{R} for metric (27) and replacing it in (28) we find:

$$6\frac{\ddot{a}(t)}{a(t)} + 2\frac{\ddot{R}(t)}{R(t)} + 6\left(\frac{\dot{a}(t)}{a(t)}\right)^2 + 6\frac{\dot{a}(t)}{a(t)}\frac{\dot{R}(t)}{R(t)} - \frac{\sqrt{21}\mu^2}{3} = 0 \quad (33)$$

by solving simultaneously (30),(31),(32),(33) we find $a(t)$ and $R(t)$:

$$a(t) = \pm \frac{\sqrt{-7\mu e^{\frac{\mu t \sqrt{5189}^{\frac{1}{4}}}{15}} \sqrt{5189}^{\frac{3}{4}} (e^{\frac{2\mu t \sqrt{5189}^{\frac{1}{4}}}{15}} C_2 - C_3)}}{\mu e^{\frac{\mu t \sqrt{5189}^{\frac{1}{4}}}{15}}} \quad (34)$$

$$R(t) = C_1 \frac{(e^{\frac{4\mu t \sqrt{53}^{\frac{3}{4}} 7^{\frac{1}{4}}}{15}} C_2^2 - C_3^2)^{\frac{1}{4}} (e^{\frac{2\mu t \sqrt{53}^{\frac{3}{4}} 7^{\frac{1}{4}}}{15}} C_2 + C_3)^{\frac{3}{4}}}{(e^{\frac{2\mu t \sqrt{53}^{\frac{3}{4}} 7^{\frac{1}{4}}}{15}} C_2 - C_3)^{\frac{3}{4}} (e^{\frac{4\mu t \sqrt{53}^{\frac{3}{4}} 7^{\frac{1}{4}}}{15}})^{\frac{1}{8}}} \quad (35)$$

In (34),(35) we have three free parameters: C_1, C_2, C_3 . In section IV we will try to determine them from the available observational parameters.

V. DENSITY AND PRESSURE FROM STM

We consider a perfect fluid energy momentum tensor for dark energy in four dimensions of the form:

$$T_\nu^\mu = \text{diag}(\rho_{DE}(t), -p_{DE}(t), -p_{DE}(t), -p_{DE}(t)) \quad (36)$$

Wesson [22] suggested that the new terms due to fifth dimension which depend on $R(t)$ (and ψ in noncompactified Kaluza-Klein cosmology) in H_0^0 and H_a^a (where $a=1, 2, 3$), are density and pressure of matter respectively. The main proposal of our model is to choose them as density and pressure of the dark energy. By using equations (30) and (31), we obtain the following expressions for $\rho_{DE}(t)$ and $p_{DE}(t)$:

$$\hat{H}_0^0 = f(a(t)) + \rho_{DE}(t) \quad (37)$$

$$\hat{H}_a^a = g(a(t)) - p_{DE}(t) \quad (38)$$

and:

$$\rho_{DE}(t) = \frac{10}{7} \frac{\ddot{R}(t)}{R(t)} \quad (39)$$

$$p_{DE}(t) = -\frac{10}{7} \frac{\dot{a}(t)}{a(t)} \frac{\dot{R}(t)}{R(t)} \quad (40)$$

It is clear that $\rho_{DE}(t)$ and $p_{DE}(t)$ vanish for a model with constant $R(t)$.

VI. TYPICAL VALUES FOR THE CONSTANTS

To specify C_2 , C_3 and μ we appeal to the observational value of some cosmological parameters. From recent observations, we know that the Hubble constant at present time is about $73 \pm 8 \text{ km s}^{-1} \text{ Mpc}^{-1}$ or in SI :

$$H(t_0) = \frac{\dot{a}(t)}{a(t)}|_{t_0} \simeq 23.6 \pm 3 \times 10^{-19} \text{ s}^{-1} \quad (41)$$

We also know that:

$$H(t) = \frac{\dot{a}(t)}{a(t)} \quad (42)$$

The other cosmological parameter is the density parameter $\Omega = \frac{8\pi G\rho}{3H_0^2}$. Recent observations suggest that the $\Omega_{tot} = \Omega_{DE} + \Omega_m \simeq 1$ and the $\Omega_{DE} \simeq 0.7$. Another cosmological parameter is the equation of state parameter (EoS) $w(t) = \frac{p(t)}{\rho(t)}$. Recent observations limit the value of ω_{DE} between -0.4 and -1.02 . Another important cosmological parameter is the transition redshift z_T , in which the cosmic evaluation transfers from the decelerated era to the accelerated era or equivalently the value of w_{DE} is getting less than -0.3 . Observations suggest that the value of z_T is between 0.15 to 0.5 [23]. We obtain these cosmological parameters for our model and by this means we can get some idea about the free parameters of the model. Here, we make five choices and discuss them separately, in order to get an idea about the behavior of the solutions. The constant C_1 can be determined by observations on variation of some parameters during the history of the Universe such as the fine structure constant α [24].

A. Choice 1: $C_2 = -1, C_3 = 0, \mu = 8.5 \times 10^{-18} s^{-1}$

In this case $w_{DE}(t)$ and $\Omega_{DE}(t)$ both are constant :

$$w_{ED}(t) = -1 \quad \Omega_{DE}(t) = 0.47 \quad (43)$$

This case corresponds to a cosmological constant, but $\Omega_{DE}(t)$ does not match with the observations. In this case $H_0 = 0.23 \times 10^{-17} s^{-1}$ in SI, which is in line with observations.

B. Choice 2: $C_2 = -1, C_3 = -1, \mu = 7 \times 10^{-18} s^{-1}$

We plot $w_{DE}(t)$ and $\Omega_{DE}(t)$ for this choice in Fig.1. In this case at $z = 0$ we have :

$$\omega_{ED}(0) = -0.55 \quad \Omega_{DE}(0) = 0.61 \quad (44)$$

Observations suggest that at the present time ($z = 0$); $\Omega_0 = \Omega_{DE0} + \Omega_{m0} \simeq 1$ so we obtain $\Omega_{m0} = 0.39$. In this case $H_0 = 0.23 \times 10^{-17} s^{-1}$ in SI, which is in line with observations. The value

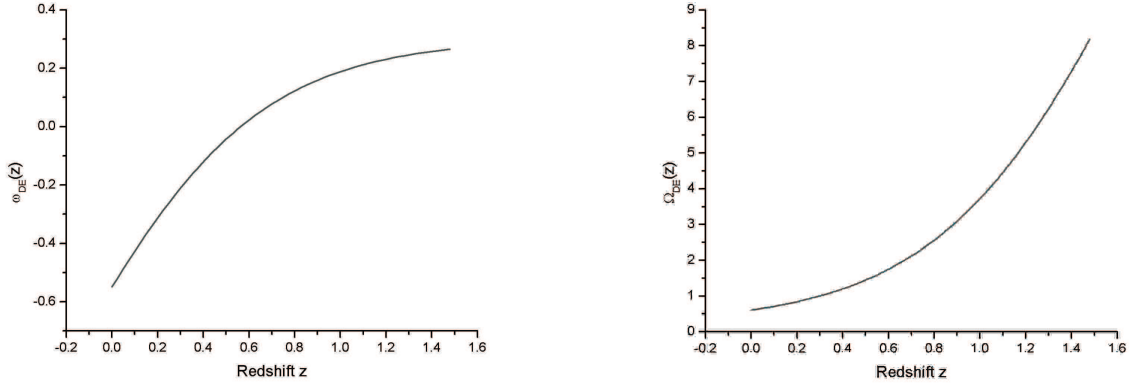


FIG. 1: $w_{DE}(t)$ (left) and $\Omega_{DE}(t)$ (right) for the case $C_2 = -1$ and $C_3 = -1, \mu = 7 \times 10^{-18} s^{-1}$

of z_T in this case is 0.2.

C. Choice 3: $C_2 = -9, C_3 = -9, \mu = 6 \times 10^{-18} s^{-1}$

We plot the $w_{DE}(t)$ and $\Omega_{DE}(t)$ in Fig.2. In this case at $z = 0$ we have :

$$w_{ED}(0) = -0.39 \quad \Omega_{DE}(0) = 0.68 \quad (45)$$

In this case $H_0 = 0.19 \times 10^{-17} s^{-1}$ in SI, which is in agreement with observations. The value of z_T in this case is 0.1.

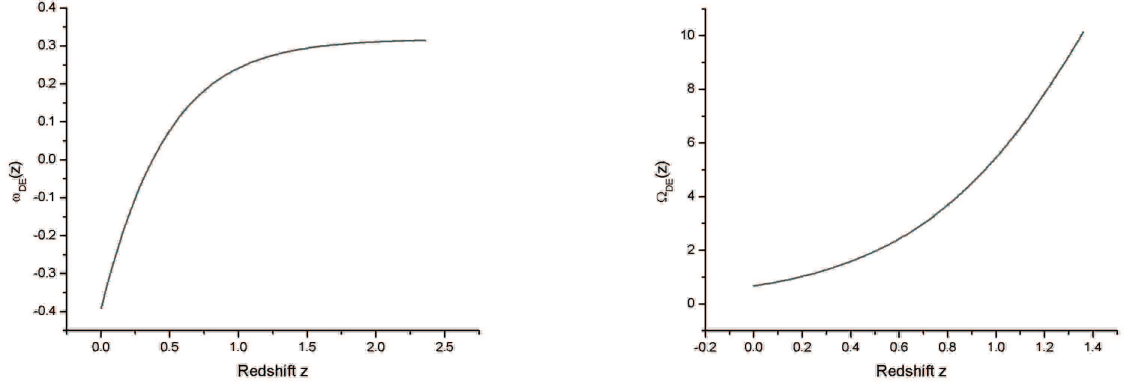


FIG. 2: $w_{DE}(t)$ (left) and $\Omega_{DE}(t)$ (right) for the case $C_2 = -9$ and $C_3 = -9, \mu = 6 \times 10^{-18} s^{-1}$

D. Choice 4: $C_2 = -3, C_3 = -6, \mu = 8 \times 10^{-18} s^{-1}$

We plot the $w_{DE}(t)$ and $\Omega_{DE}(t)$ in Fig.3. In this case at $z = 0$ we have :

$$w_{ED}(0) = -0.47 \quad \Omega_{DE}(0) = 0.64 \quad (46)$$

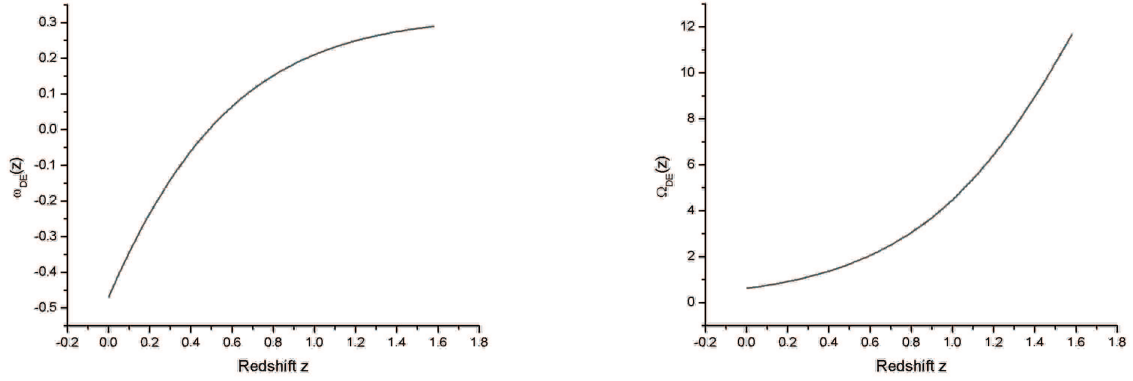


FIG. 3: $w_{DE}(t)$ (left) and $\Omega_{DE}(t)$ (right) for the case $C_2 = -3$ and $C_3 = -6, \mu = 8 \times 10^{-18} s^{-1}$

In this case $H_0 = 0.24 \times 10^{-17} s^{-1}$ in SI, which is in line with observations. The value of z_T in this case is 0.15.

E. Choice 5: $C_2 = -4, C_3 = -9, \mu = 7 \times 10^{-18} s^{-1}$

We plot the $w_{DE}(t)$ and $\Omega_{DE}(t)$ in Fig.4 . We have at $z = 0$:

$$w_{ED}(0) = -0.25 \quad \Omega_{DE}(0) = 0.75 \quad (47)$$

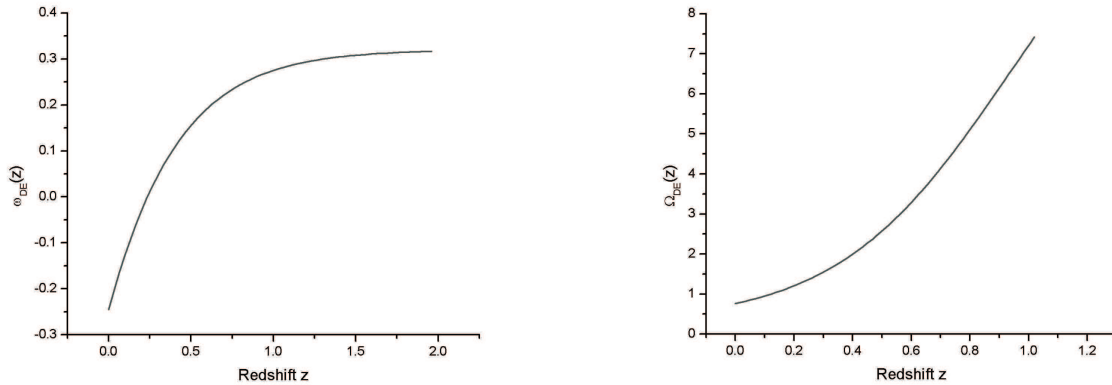


FIG. 4: $w_{DE}(t)$ (left) and $\Omega_{DE}(t)$ (right) for the case $C_2 = -4$ and $C_3 = -9, \mu = 7 \times 10^{-18} s^{-1}$

In this case $H_0 = 0.24 \times 10^{-17} s^{-1}$ in SI, which is in agreement with observations and the value of w_{DE} at the present time is more than -0.3 , so we don't obtain an accelerating universe.

VII. CONCLUSION

In this paper, we proposed a new model for dark energy by using the CDTT $f(R)$ gravity model and applying the STM formalism to a five dimensional metric and interpret the fifth dimension as dark energy source. Then we used some cosmological parameters and adjusted the model with observations to find the typical values for the free parameters of the model (C_2 , C_3 and μ). When we want to approach $\Omega_{DE} = 0.7$ the equation of state parameter tends to less than $-1/3$. In the forth case considered, the cosmological parameters are close to the results of observations, although this does not lead to a unique choice of the parameters. If we take $C_3 = 0$, the model reduces to the cosmological constant model.

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